Optimal Control of a Ballbot

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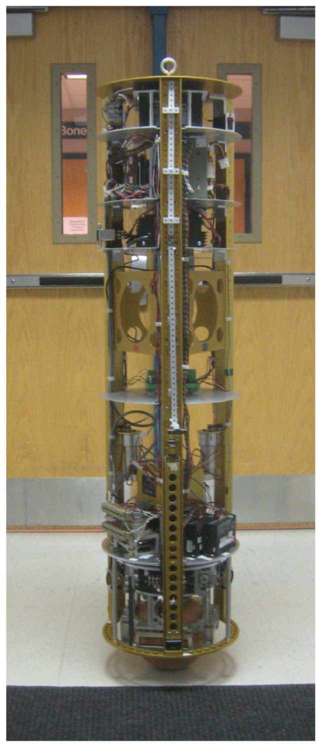

Figure 1: Ballbot, from [2, 1].

1 What is the Ballbot?

The ballbot (see Figure 1) form is tall and narrow, and it moves on a single spherical wheel - which allows it **omnidirectional** movement capacity, making it ideal for movement in cluttered urban environments like our homes or commercial establishments. However, this also gives it a high center of gravity and a small footprint, making the ballbot statically unstable. The ballbot is heavily underactuated, since its dynamics are constrained, we can't directly control pitch and roll of the bot.

This motivates my study of control algorithms for this robot for the course project.

2 Dynamics of the Ballbot

2.1 Planar Model

We make the following assumptions about the ballbot:

1. It is modeled as a rigid cylinder on top of a rigid sphere (see Fig. 2a).

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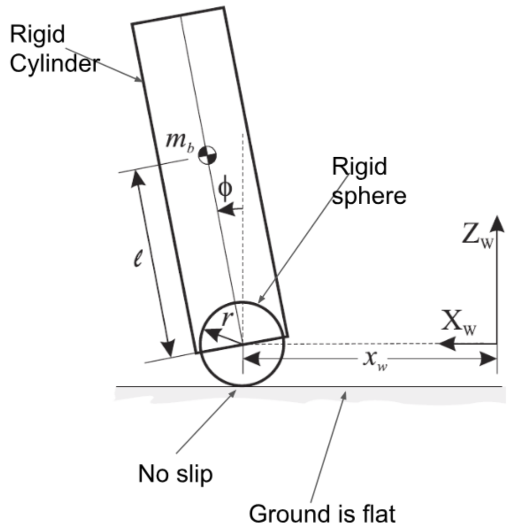
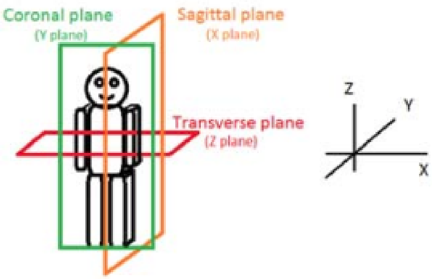


(a) Planar Model of the Ballbot



(b) Coronal and Sagittal plane (explainer)

2. Since the ballbot is constructed to be symmetrical and have identical moments of inertia along both X and Y axes, we can model it using a planar model.
3. This also allows us to assume that motion in the sagital and coronal plane is decoupled and identical (see Fig. 2b).
4. There is no slip between the wheel and the floor
5. The ground is flat

Given these assumptions, we can design two (almost) independent planar trajectory controllers:

1. One to control pitch and X-displacement
2. One to control roll and Y-displacement

Hence going forward for the rest of this report, we will only think about control and planning in one plane - it will be identical in the other plane. Note that I ignore yaw throughout - it is of no use for trajectory planning since the ballbot is omnidirectional anyway.

2.2 Deriving the dynamics model

2.2.1 List of Variables

T : Kinetic Energy

V : Potential Energy

L : Lagrangian $L = T \quad V$.

m_b : mass of body (rigid cylinder)

I_b : moment of inertia of body through its center of mass

l_b : distance of center of mass of body from ground

m_w : mass of wheel

r_w : radius of wheel

I_w : moment of inertia of wheel through its center of mass

: body angle, see Fig. 2a: defined as angle between vertical and body axis

: angle of the wheel, chosen such that the horizontal position of the ball x_w w.r.t the world frame is given by $x_w = r_w(\quad + \quad)$

q : generalized coordinate vector = $[\quad]^T$

2.2.2 Derivation from energy

Assuming potential energy to be zero at the level of the wheel's center, the system's potential energy is

$$V = m_b g(l_b \cos \quad + r_w) \tag{1}$$

and its kinetic energy is given by the sum of translational and rotational kinetic energy of the wheel + body.

$$\begin{aligned} T = & \frac{(I_w + m_w r_w^2)\dot{\ }^2}{2} + \frac{(I_b + m_b l_b^2)\dot{\ }^2}{2} \\ & + \frac{m_b[r_w^2(\dot{\ }^2 + \dot{\ }^2 + 2\dot{\ }\dot{\ }) + l_b^2\dot{\ }^2 + 2r_w l_b(\dot{\ }\dot{\ }^2 + \dot{\ }^2)\cos \quad]}{2} \end{aligned} \tag{2}$$

With Eqns. 1,2 and the Euler-Lagrange equation:

$$\frac{d}{dt}\frac{@L}{@\dot{q}} \quad \frac{@L}{@q} = 0 \tag{3}$$

we get the following equations of motion for the ballbot:

$$\begin{bmatrix} & + \cos \\ + \cos & + \ + 2 \ \cos \end{bmatrix} \ddot{q} + \begin{bmatrix} \sin \ \dot{\ }^2 \\ \sin \ \dot{\ }^2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{g \sin}{r} \end{bmatrix} = \begin{bmatrix} \\ 0 \end{bmatrix} \quad \begin{bmatrix} D_c sgn(\dot{\ }) + D_c \dot{\ } \\ 0 \end{bmatrix} \tag{4}$$

Going forward we consider state

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \\ \\ \dot{\ } \\ \dot{\ } \end{bmatrix} \tag{5}$$

3 Open Loop Trajectory Planning: For the Planar Ballbot

Desiderata:

1. Go from an initial position in the X-plane to a final desired position, both starting and ending from rest and in a balanced position

$$x_i = \begin{bmatrix} _i \\ 0 \\ 0 \\ 0 \end{bmatrix}; x_d = \begin{bmatrix} _d \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

2. Do this in minimum time, and with minimum control effort

The following parametric trajectory is chosen for the body angle, to account for the fact that for the ballbot to move forward and then come to rest, it has to lean forward first and then back to stop (see Fig. 3):

$$_p(t) = \ _{pa_1} sech(k\frac{2t \quad t_0 \quad t_m}{t_m \quad t_0}) + \ _{pa_2} sech(k\frac{2t \quad t_f \quad t_m}{t_f \quad t_m}) + \ _{pa_0} \tag{7}$$

with $t_m = (t_0 + t_f)=2$ and $k = 9$. $_{pa_1};\ _{pa_2}; t_f$ are parameters that can be optimized. $_{pa_0}$ is chosen such that $_p(t_0) = 0$ and $_p(t_f) = 0$.

This trajectory parameterization ensures that $_p(t_0) = 0,\ _p(t_f) = 0,\ \dot{}_p(t_0) = 0,\ \dot{}_p(t_f) = 0$

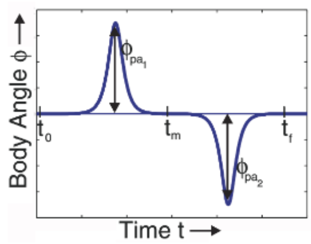


Figure 3: Plot of the parameterized trajectory of body angle vs time

Now we formulate a cost function which imposes soft constraints to ensure that the ballbot reaches the desired final position and comes to rest, along with penalizing time and control effort expended to get there.

$$J = w_1(\ (t_f) \quad _d)^2 + w_2(\dot{}^2) + w_3(t_f \quad t_0)^2 + w_4 \int_{t_0}^{t_f} {}^2 dt \tag{8}$$

Hence the optimization problem becomes:

$$\min_{pa_1;\ pa_2;t_f} J \tag{9}$$

subject to dynamics constraints from Eqn 4.

This is solved using `fminsearch` in MATLAB, and the dynamics can be resolved using `ode45`.

3.0.1 Results

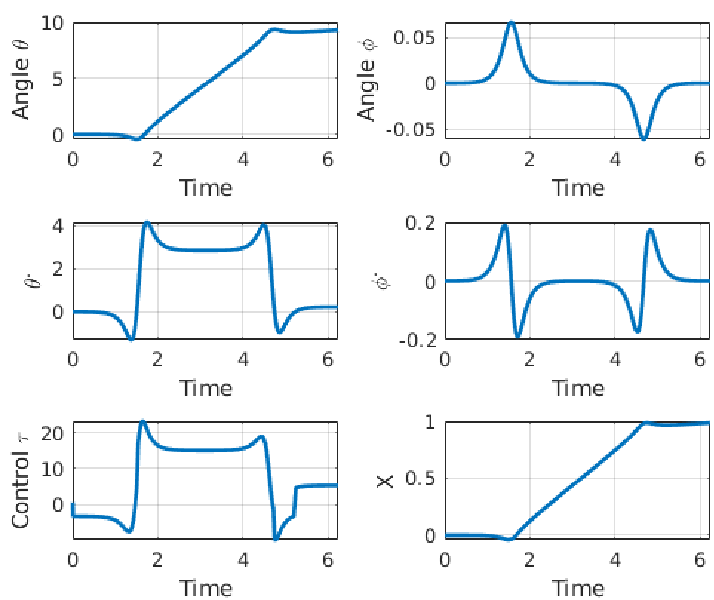


Figure 4: Open Loop Trajectory Planning to take the ballbot from $x = 0$ to $x = 1$. Plots are (L-R,top to bottom) , , $\dot{}$, $\dot{}$, , X vs time t. Video at `https://goo.gl/taQ36R`.

4 Open Loop Trajectory Planning: For the 3-D Ballbot

Now we consider the full 3-dimensional case when the ballbot moves along the 2-dimensional floor (XY-plane). As mentioned earlier, the ballbot is completely symmetrical and we assume that motion in the sagittal and coronal plane is decoupled and identical. However, trajectory planning cannot be done completely independently because **time** is still a optimizing variable being shared by both spatial dimensions, i.e., to reach a desired configuration on the floor, the time t_f must be the same for body angle trajectories on the floor. So now the optimization problem becomes:

$$\min_{pa_1^x;\ pa_2^x;\ pa_1^y;\ pa_2^y;t_f} J$$
$$= w_1(\ \ (t_f)\ \ \ \ _d)^2 + w_2(\_^2) + w_3(t_f\ \ \ t_0)^2 + w_4 \int_{t_0}^{t_f} (\ _x^2 +\ _y^2)dt \qquad (10)$$

with 2 sets of dynamics constraints from Eqn. 4.

4.0.1 Results

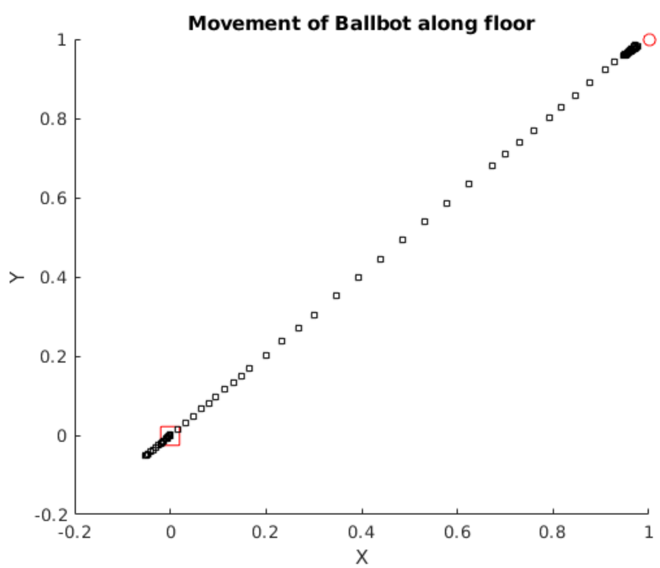


Figure 5: Movement of Ballbot along floor from (0,0) to (1,1). See video at `https://goo.gl/zrCKAS`

References

[1] Ralph Hollis. Ballbots. *Scientific American*, 295(4):72–77, 2006.

[2] Umashankar Nagarajan, George Kantor, and Ralph L Hollis. Trajectory planning and control of an underactuated dynamically stable single spherical wheeled mobile robot. In *Robotics and Automation, 2009. ICRA'09. IEEE International Conference on*, pages 3743–3748. IEEE, 2009.